Edge-Dominance Games on Graphs

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Motivation

How should players act in competitions where positions possess advantages over others?

How does a bounded capability to change actions between rounds influence optimal strategies?

What should players do in these situations?

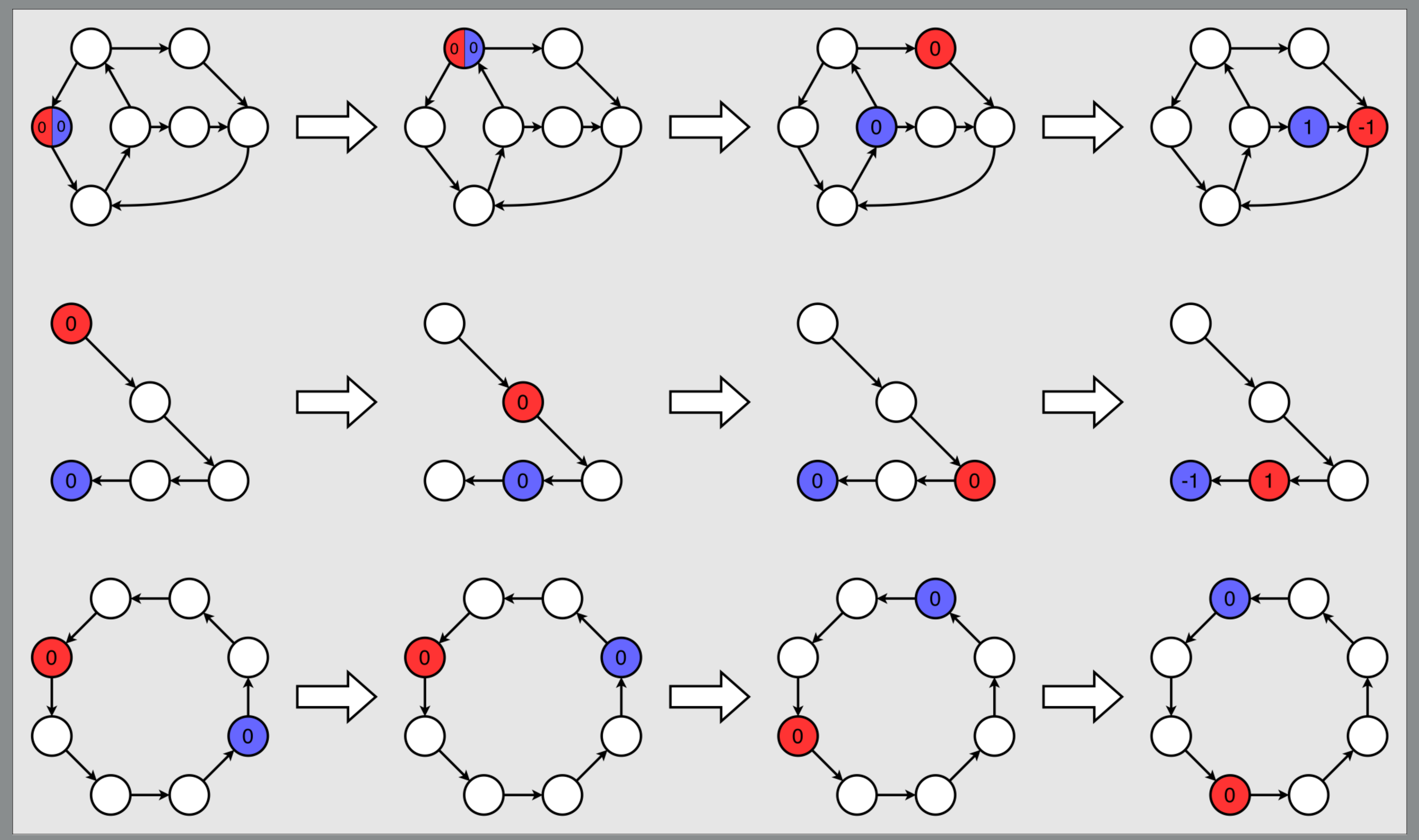
Examples from the real world:
- Physical conflict
- Duopoly markets
- Hotelling models

Model

A repeated game. Two players move simultaneously on the nodes of a graph. They get payoff in every round:
- If one player is at a parent of the other player's node, the player at the parent node gets payoff 1 and the player at the child node gets payoff -1.
- If the players occupy the same node or their distance is ≥ 2, then they both get 0 payoff.

Note: the edges are undirected for movement.
The game ends at a given time step with constant probability 1-δ.

Examples (numbers = payoffs)



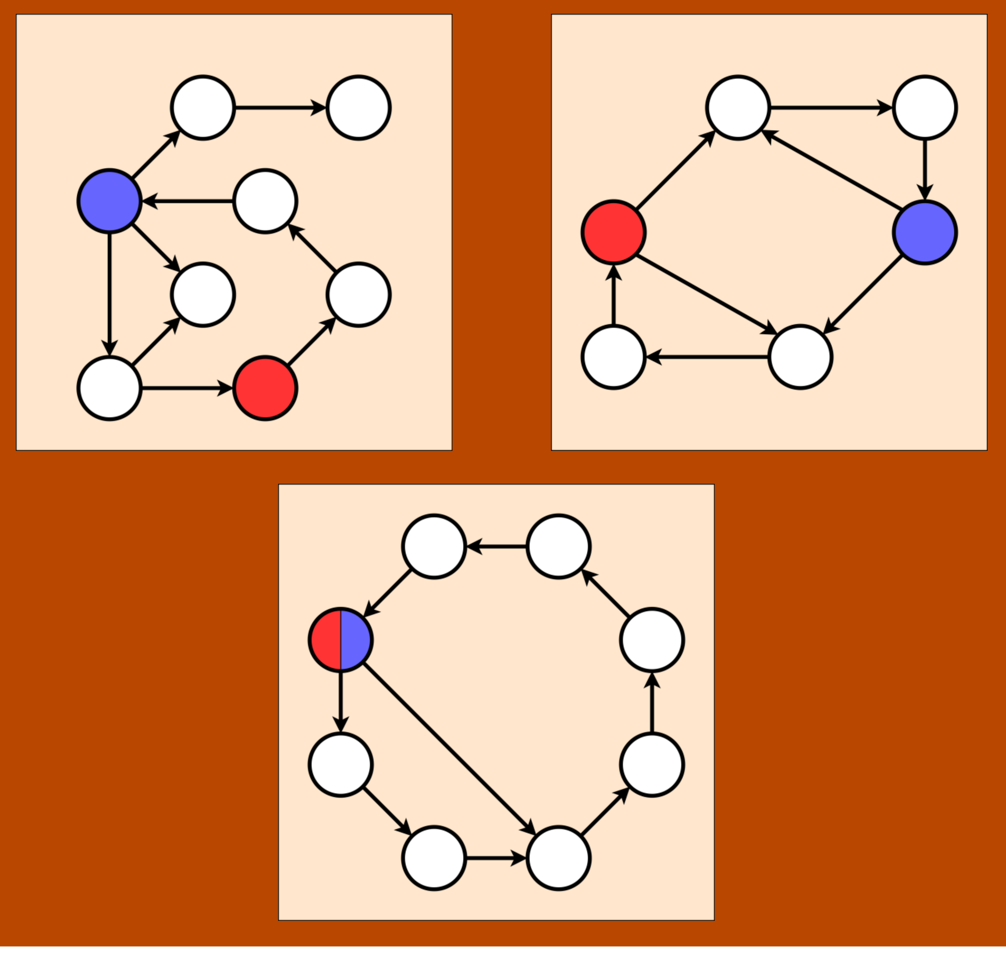
Pure Nash Equilibria

3 natural types of 0-payoff, pure Nash equilibria (PNEs):

1. Static equilibrium: the players stay put at their nodes.
2. Walking together (WT) equilibrium: the players jointly occupy a node and walk together on the graph.
3. Chase equilibrium: one player chases the other player on the graph from a distance ≥ 2.

All these equilibria yield 0 payoff for both players.

PNE examples:



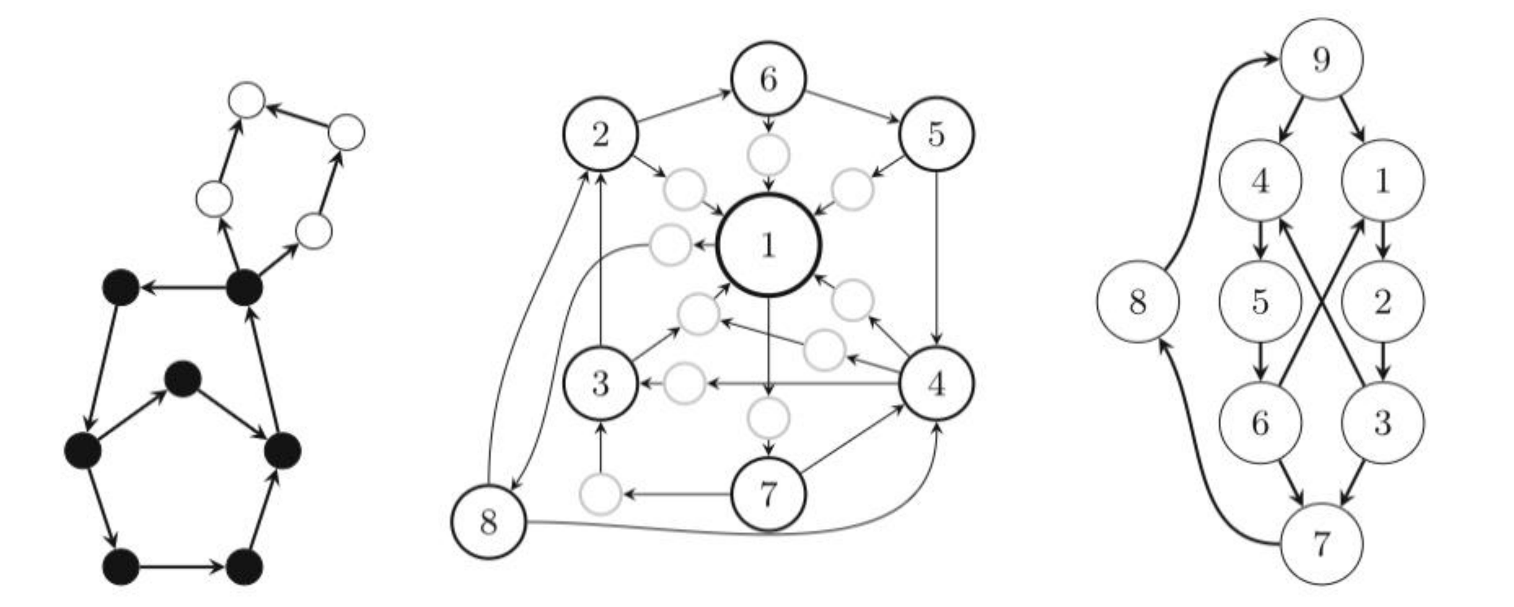
When does a player have a winning strategy? When can both players force a tie?

Theorem: in a graph of girth ≥ 6, one player has a winning strategy iff it is initially at a parent of the other player. The achievable payoff for the winner in optimal play is in the range [1-δ, 4(1-δ)/(4-δ)].
If the two players are initially at distance 0 or ≥ 2 from each other, then they can both force a tie.
Static, WT and chase equilibria all exist.

Theorem: in a graph of girth ≥ 4 without *trapping cycles*, player 1 has a winning strategy over player 2 iff in the *tree decomposition*, the shortest path between the players favors player 1 and player 2 does not have safe components on the way.

Theorem: in an *outerplanar graph* (a planar graph where all nodes touch the outer face) of girth ≥ 5 both players can force a tie via both WT and chase PNEs.

What if there are trapping cycles? Then neither static, WT nor chase equilibria can be guaranteed to exist:



(a) A graph with no static equilibria (b) A graph with no WT equilibria (c) A graph with no 2-chase equilibria

Comparison to cops and robbers

In cops and robbers games, a predefined group of cops try to catch a predefined robber on a graph (reach the robber's location), while the robber tries to evade the cops indefinitely.

Two main differences with our game:
1. In our game, either player can win payoff against the other player. The advantages in the game are inherent to the environment rather than the players' roles.
2. In our game, a player can only gain payoff by reaching a superior position to the other players'.