Multi-marginal Schrödinger Bridges with Iterative Reference Refinement

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Motivating example: study dynamics of mRNA concentration for cancer treatments

- Idea: model mRNA concentration in each cell using a Stochastic Differential Equation (SDE)
- Issue 1: cannot measure mRNA concentration in continuous time
- Issue 2: we destroy cell to measure mRNA concentration → cannot track trajectory of any individual cell

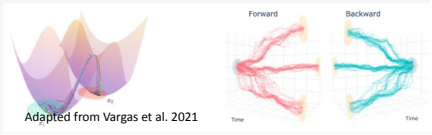
We introduce a new method to learn dynamics given only sparse time snapshots of data without individual trajectories. We leverage an iterative projection mechanism inspired by Schrödinger bridges.

Can you using existing methods?

- **SDE literature**: assume trajectories densely sampled in time → not applicable to data like mRNA example
- **Potential solution:** Schrödinger Bridges (SBs)
- **Issue 1**: focus on dynamics between two time points → what if we have access to multiple time snapshots and want to account for long-term dependencies?
- **Issue 2**: SBs require a single fixed reference dynamic → what if we have some knowledge about the problem?

Schrödinger Bridges

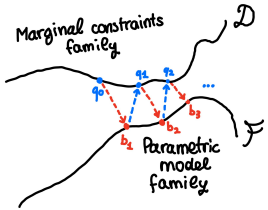
- Task: find a pair of forward-backward SDEs that interpolate between two distributions, ensuring that the learned dynamics are as close as possible to a predefined reference dynamics



Adapted from Vargas et al. 2021

An iterative approach

- Iterative between interpolation and reference refinement
- Take advantage of structure in model family
- converges to the best estimator minimizing the KL divergence Between interpolation and References



Testing our method

Baselines

Vanilla multimarginal SB: learn an SDE for each pair of adjacent time steps (with Brownian motion reference)

Deep Momentum multi marginal SB: learn trajectory of underlying velocity field (instead of location field) to get smooth trajectories

Data

Lotka-Volterra: classic parametric dynamical system

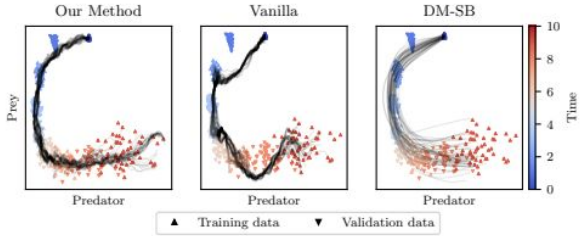
Repressilator: model capturing mRNA levels of three genes that cyclically suppress each other's synthesis

Gulf of Mexico: real ocean current in GoM

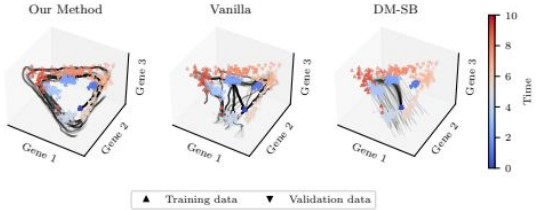
Experimental details

We train the models on data from even steps (starting at 0). Odd steps data are holdout validation data

Lotka-Volterra



Repressilator



Gulf of Mexico